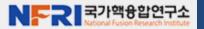


EXTENDED NEOCLASSICAL PLASMA ROTATION AND TRANSPORT THEORY

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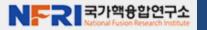
Atlanta International School, Atlanta, GA



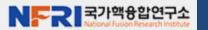


CONTENTS

- 1. EXTENDED ROTATION THEORY (ERT)
- 2. MAJOR DEVELOPMENTS
- 3. ONGING RESEARCHES
- 4. CONCLUSIONS
- 5. Q&A
- 6. APPENDICES
 - A. GENERALIZED VISCOUS EFFECTS FOR NON-AXISYMMETRIC TOKAMAK PLASMAS
 - B. A COUPLED STUDY OF PLASMA ROTATION AND TRANSPORT
 - C. EXPERIMENTAL EXPERIENCES



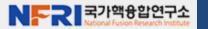
Extended Plasma Rotation Theory(ERT)?





BACKGROUND

- Plasma rotation theories developed by two main approaches
 - Hirshman-Sigmar approach
 - Hirshman and Sigmar, Nuclear Fusion 21 (1981)
 - Most recent publication: Houlberg et al., 1998
 - most famous with two (parallel/perpendicular) Momentum Balance
 Equations(MBE) to calculate neoclassical rotations of multi-ions and Er
 - Stacey-Sigmar approach
 - Stacey and Sigmar, Phys. Fluids 28, 2800(1985)
 - Most recent publication: Bae et al., Nuclear Fusion 2013
 - introduced as "Extended Plasma Rotation Theory"
 - Decomposes MBE in three coordinates (radial, poloidal, toroidal)
 - direct comparisons with Vt and Vp measurements possible
 - Radial transport calculations in radial coordinates
 - Others





BACKGROUND

Neoclassical plasma rotation codes

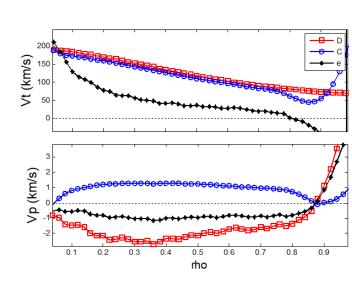
NCLASS

- based on Hirshman-Sigmar approach
 - Publication: Houlberg et al., Phys. Plasmas, 4 (1997)
- calculate neoclassical rotations of multiions
- Embedded in TRANSP

GTROTA

- based on Stacey-Sigmar approach
 - Publication: Bae et al., Comp. Phys. Comm. (2013)
- Uses D-shaped Miller flux surface geometry
 - Miller et. al., Phys. of Plasmas, 5 (1998)
- calculates rotation velocities up to four ion species and electron
- A non-linear iteration code in Matlab





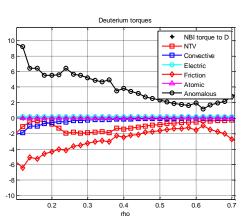




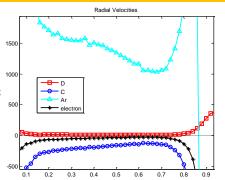
BACKGROUND

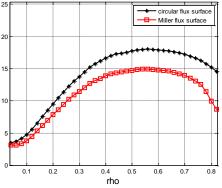
GTROTA major outputs

- Vt and Vp rotation velocities of 4 ion, species and electron (previous slide)
- Radial velocity (Vr) and electric field (Er)
- All the torques in toroidal angular torque balance
 - Including viscous torques
- Poloidal in-out / up-down asymmetries
 - in density, velocity, and electrostatic potential

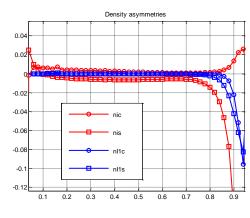


 Vt calculations with DIII-D, KSTAR, and EAST shots agree within 15% of carbon or Ar measurements (CES and XICS measurements) for rho < 0.85





Er(kV/m)







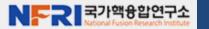
- Based on collisionality-extended Braginskii's closure
 - Meaning that it is based on Braginskii's closure but extended for arbitrary collisionality
 - Details in the next slide
 - Extension to Mikhailovski-Tsyin's closure in progress [Plasma Phys 13, 785 (1971)]
 - For better accuracy in the edge Rotation Study (rho 0.85 to 1.0)

$$\boxed{-\Omega_a \hat{b} \times \vec{V}_a = \partial_t \vec{V}_a + \vec{V}_a \cdot \nabla \vec{V}_a + \frac{1}{m_a n_a} \nabla \cdot \vec{\pi}_a + \frac{1}{m_a n_a} \nabla p_a - \frac{q_a}{m_a} \vec{E} - \frac{1}{m_a n_a} \vec{C}_a^{10}}$$

$$-\Omega_{a}\left(\overrightarrow{\pi}_{a}\times\hat{b}-\hat{b}\times\overrightarrow{\pi}_{a}\right)\approx\frac{2p_{a}\overrightarrow{\nabla V}_{a}}{2p_{a}\overrightarrow{\nabla V}_{a}}+\frac{4}{5}\overrightarrow{\nabla q_{a}}+higher order terms$$

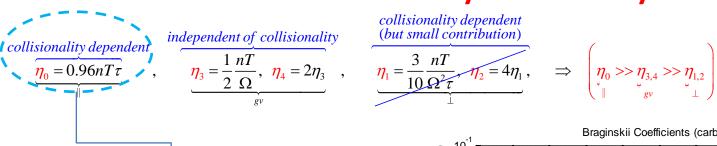
$$Mikhailovski-Tsypin$$

$$-\Omega_{a}\left(\hat{b}\times\overrightarrow{q_{a}}\right)=\frac{5}{2m}p_{a}\nabla T_{a}+\frac{1}{2}\overrightarrow{C}_{a}^{11}+O\left(higher\right)$$



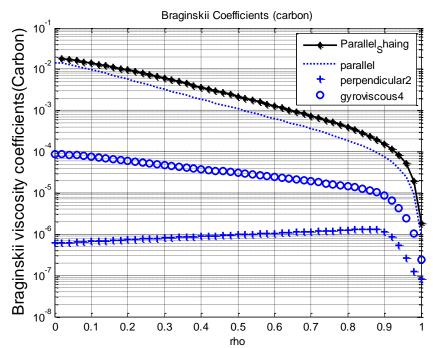


- Collisionality-extended Braginskii's viscosity representation
 - Parallel viscosity coefficient extended to low collisionality (trapped particle effect) by Shaing
 - -> Calculations valid for arbitrary collisionality



Shaing-banana plateau-PS:

$$\eta_{0j} = \frac{n_{j} m_{j} V_{thj} q R_{0} \varepsilon^{-3/2} v_{jj}^{*}}{\left(1 + \varepsilon^{-3/2} v_{jj}^{*}\right) \left(1 + v_{jj}^{*}\right)} \equiv n_{j} m_{j} V_{thj} q R f_{j}$$

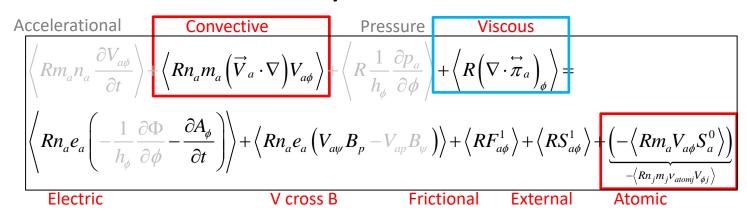






Retains all the terms in Toroidal and Poloidal MBE

- Except vanishing ones due to equilibrium and axisymmetry (in grey)
- Includes Reynolds Stress terms (Convective and Atomic torques)
- Atomic term calculated with TRANSP
- No gyroviscous cancellation assumed
- Numerical model becomes extremely non-linear



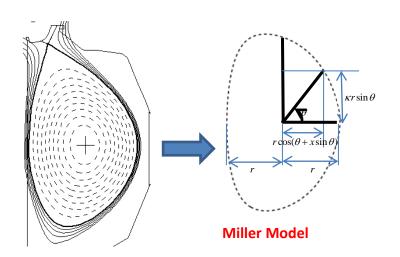
$$\left\langle rm_{a}n_{a}\frac{\partial V_{a\theta}}{\partial t}\right\rangle + \left\langle rn_{a}m_{a}\left(\overrightarrow{V}_{a}\cdot\nabla\right)V_{a\theta}\right\rangle + \left\langle r\frac{1}{h_{\theta}}\frac{\partial p_{a}}{\partial\theta}\right\rangle + \left\langle r\left[\nabla\cdot\overrightarrow{\pi}_{a}\right]_{\theta}\right\rangle =$$

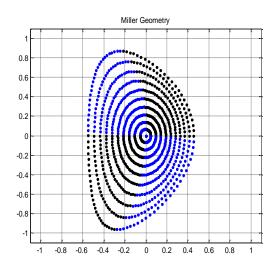
$$\left\langle r\left(-\frac{1}{h_{p}}\frac{\partial\Phi}{\partial\theta} - \frac{\partial A_{\theta}}{\partial t}\right)\right\rangle + \left\langle rn_{a}e_{a}\left(V_{a\phi}B_{r} - V_{ar}B_{\phi}\right)\right\rangle + \left\langle rF_{a\theta}^{1}\right\rangle + \left\langle r\left(S_{a\theta}^{1} - m_{a}V_{a\theta}S_{a}^{0}\right)\right\rangle$$





applies D-shaped flux surface geometry using Miller model [Miller et. al., Phys. of Plasmas, 5 (1998)] with Shafranov shifts





- Flux surface averages (FSAs) and geometric scale factors(h's) for Miller geometry
 - Example: Er formula below shows FSAs and scale factors (h)

$$\overline{E}_{r} = -V_{thj}\overline{B}_{\theta} \left[V_{\theta j} \frac{\left\langle \frac{1}{1 + \varepsilon \cos \xi} \right\rangle}{\left\langle \frac{1}{h_{r}} \right\rangle} - V_{\phi j} \left(1 + \frac{\partial R_{0}(r)}{\partial r} \right) \frac{\left\langle \frac{1}{(1 + \varepsilon \cos \xi)} \frac{1}{h_{r}} \right\rangle}{\left\langle \frac{1}{h_{r}} \right\rangle} - \frac{1}{V_{thj}} \frac{1}{\overline{n_{j}} e_{j} \overline{B}_{\theta}} \frac{\partial \overline{P}_{j}}{\partial r} \right]$$





Up-down asymmetry

In-out asymmetry

EXTENDED ROTATION THEORY

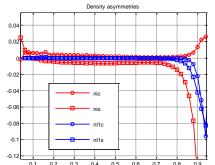
- Includes first-order poloidal perturbations (poloidal asymmetries) in density, velocity, and electrostatic potential
 - Represented with the 1st order Fourier series

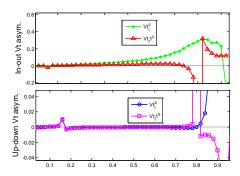
$$X_{j}(\rho,\phi) \approx \overline{X}_{j}(\rho) \left[1 + \sum_{n=1}^{\infty} \left(\underbrace{X_{j}^{nc}(\rho) \cos(n\phi)}_{in-out \ asymmetry} + \underbrace{X_{j}^{\alpha s}(\rho) \sin(n\phi)}_{up-down \ asymmetry} \right) \right]$$

- Sine function representing up-down asymmetry
- Cosine function representing in-out asymmetry



- Bae et al., Nuclear Fusion 2013
- Bae et al., Phys of Plasmas, 2014
- A draft under review





- Georgia Tech Fusion Research Center has recently developed a code to calculate rotations with 10th order perturbations for accuracy
 - R. King, MS thesis, Georgia Institute of Technology, May 2019





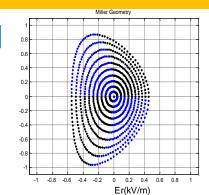
- Radial transport of all ion species calculated
 - Er calculated self-consistently

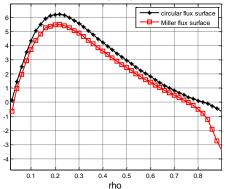
$$\overline{E}_{r} = -V_{thj}\overline{B}_{\theta} \left[V_{\theta j} \frac{\left\langle \frac{1}{1 + \varepsilon \cos \xi} \right\rangle}{\left\langle \frac{1}{h_{r}} \right\rangle} - V_{\phi j} \left(1 + \frac{\partial R_{0}(r)}{\partial r} \right) \frac{\left\langle \frac{1}{(1 + \varepsilon \cos \xi)} \frac{1}{h_{r}} \right\rangle}{\left\langle \frac{1}{h_{r}} \right\rangle} - \frac{1}{V_{thj}} \frac{1}{\overline{n_{j}} e_{j} \overline{B}_{\theta}} \frac{\partial \overline{P}_{j}}{\partial r} \right]$$

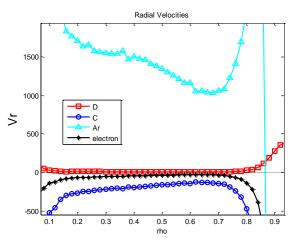
- Calculation with Miller flux surfaces published in 2014
 - Bae et al., Phys of Plasmas, 2014
 - Difference in Er with Circular vs. Miller model published

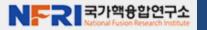
$$\overline{E}_{r}^{cir} = \frac{1}{\overline{n_{j}}e_{j}} \frac{\partial \overline{P}_{j}}{\partial r} - \left[V_{\theta j} \overline{B}_{\phi} - V_{\phi j} \overline{B}_{\theta} \right]$$

- Recent investigations on Vr and Radial flux of all ion species
 - calculated non-self-consistently
 - GTROTA updated to investigate these calculations >
 - Collaboration with other researches based on radial diffusivity coefficients (later slides)



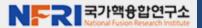






Future directions of ERT and GTROTA (mostly for edge rotation study)

Advises/ideas/recommendations welcomed!





- Extension of ERT to Mikhailovskii-Tsypin's closure
 - Include Heat Equation
 - Important for edge rotation/transport study
 - Viscosity evolution equation:

$$-\Omega_{a}\left(\hat{b}\times\vec{\pi}_{a}-\vec{\pi}_{a}\times\hat{b}\right)=2p_{a}\nabla\vec{V}_{a}+\frac{4}{5}\nabla\vec{h}_{a}-\vec{C}_{a}^{20}+higher\ orders$$

Heat flow evolution equation:

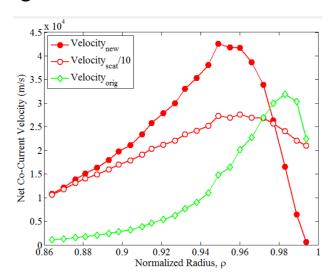
$$-\Omega_a \left(\hat{b} \times \vec{h}_a \right) = \frac{5}{2m_a} p_a \nabla T_a + \frac{1}{2} \vec{C}_a^{11} + higher \ orders$$

- > This development in progress (2016 ongoing)
 - 1ST order Fourier expansion of temperature being applied
 - This is next in line for the future development of ERT and GTROTA
 - doable within a few years to finalize the numerical model and implement it in GTROTA





- To add more edge physics including intrinsic rotation
 - Intrinsic co-current deuterium rotation due to Ion Orbit Loss (IOL)
 - Stacey, Phys of Plasmas 25, 122506 (2018)
 - The predominant IOL of CTR-current ions leaves a predominantly COcurrent edge intrinsic rotation



- This effort shouldn't take long but will need to add other edge rotation mechanisms for higher accuracy
 - Ideas/suggestions welcomed



- Develop an NTV theory based on Stacey-Sigmar approach
 - By considering non-axisymmetry in the formalism
 - General non-axisymmetric formalism published: Stacey and Bae, PoP (2015)

$$n_{j}m_{j}\left\langle \left[R\left(\overrightarrow{V}_{j}\bullet\nabla\right)\overrightarrow{V}_{j}\right]_{\phi}\right\rangle + \left\langle \left[R\left(\nabla\bullet\overrightarrow{\pi}_{j}\right)\right]_{\phi}\right\rangle = \left\langle Rn_{j}e_{j}\mathcal{E}_{\phi}^{A}\right\rangle + \left\langle Rn_{j}e_{j}V_{rj}B_{\theta}\right\rangle + \left\langle RF_{\phi j}\right\rangle + \left\langle RM_{\phi j}\right\rangle - \left\langle Rn_{j}m_{j}V_{atom j}V_{\phi j}\right\rangle$$

$$\left\langle R\left(\nabla\cdot\overrightarrow{\pi}\right)_{\phi}\right\rangle = \left\langle R^{2}\nabla\phi\cdot\left(\nabla\cdot\overrightarrow{\pi}\right)\right\rangle = \left\langle \frac{1}{Rh_{p}}\frac{\partial\left(R^{2}h_{p}\pi_{\psi\phi}\right)}{\partial l_{\psi}}\right\rangle$$

$$\left\langle n_{j}m_{j}V_{atom j}V_{\phi j}\right\rangle$$

$$\left\langle R\left(\nabla\cdot\overrightarrow{\pi}\right)_{\phi}\right\rangle = \left\langle R^{2}\nabla\phi\cdot\left(\nabla\cdot\overrightarrow{\pi}\right)\right\rangle = \left\langle \frac{1}{Rh_{p}}\frac{\partial\left(R^{2}h_{p}\pi_{\psi\phi}\right)}{\partial l_{\psi}}\right\rangle$$

 $\begin{array}{c|c}
\eta_0 >> \eta_{3,4} >> \eta_{1,2} \\
\parallel & gv & \bot
\end{array}$

Dedicated presentation slides in Appendix A

$$\left\langle R \left(\nabla \cdot \overset{\hookrightarrow}{\pi} \right)_{\phi} \right\rangle = \left\langle R^{2} \nabla \phi \cdot \left(\nabla \cdot \overset{\hookrightarrow}{\pi} \right) \right\rangle = \left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p} \boldsymbol{\pi}_{\psi\phi}^{0} \right)}{\partial l_{\psi}} \right\rangle + \left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p} \boldsymbol{\pi}_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle + \left\langle \frac{1}{Rk_{p}} \frac{\partial \left(R^{2}h_{p} \boldsymbol{\pi}_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle + \left\langle \frac{1}{Rk_{p}} \frac{\partial \left(R^{2}h_{p} \boldsymbol{\pi}_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle$$

$$axisymmetric$$

$$non-axisymmetric$$

$$oxed{\pi_{\psi\phi}^0 = -\eta_0 W_{\psi\phi}^0} oxed{W_{\psi\phi}^0}$$

$$W_{\psi\phi}^{0} = \frac{3}{2} f_{\psi} f_{p} H^{0}$$

$$f_{\psi}\equiv rac{B_{\psi}}{B}\,, \quad f_{p}\equiv rac{B_{p}}{B}\,, \quad f_{\phi}\equiv rac{B_{\phi}}{B}\,$$



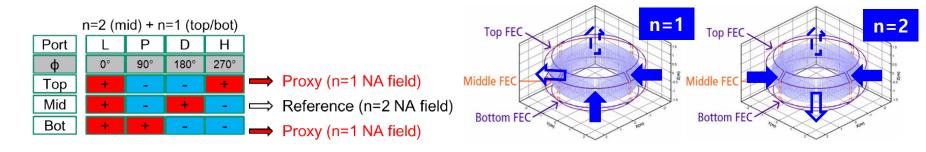


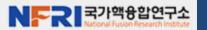
- Develop an NTV theory based on Stacey-Sigmar approach
 - Next step is to develop a numerical model for GTROTA

$$\boxed{\pi_{\psi\phi}^0 = -\eta_0 W_{\psi\phi}^0} \qquad W_{\psi\phi}^0 = \frac{3}{2} f_{\psi} f_p H^0 \qquad \boxed{f_{\psi} \equiv \frac{B_{\psi}}{B}, \quad f_p \equiv \frac{B_p}{B}, \quad f_{\phi} \equiv \frac{B_{\phi}}{B}}$$

- Question is on how to best represent magnetic perturbations
 - Up to the 4th order Fourier series being considered but still contemplating
 - Ideas being formulated from my experimental experiences at KSTAR
 - as a team member for the ELM suppression investigation using RMP coils at KSTAR
 - Publication: Jayhyun Kim et al., Nucl. Fusion 57, 022001 (2017)

$$\overrightarrow{B}_{\text{total}} = \overrightarrow{B}_{\text{eq}} + \overrightarrow{B}_{\text{int}} + \overrightarrow{B}_{\text{RMP}} + \overrightarrow{B}_{\text{plasma response}}$$





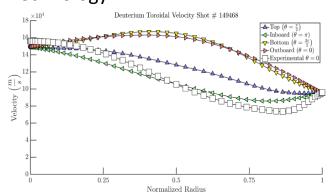


- Investigate the converging order of poloidal asymmetries
 - Georgia Tech Fusion Research Center has calculated rotations with poloidal asymmetries up to 10th order
 - Publication: R. King, MS thesis, Georgia Institute of Technology

$$n = n_0 + \sum \sum a_{ij} J_i (\lambda_{ij} \rho) \cos(j\theta) + b_{ij} J_i (\lambda_{ij} \rho) \sin(j\theta)$$

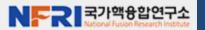
$$v_t = v_{t0} + \sum \sum a_{ij} J_i (\lambda_{ij} \rho) \cos(j\theta) + b_{ij} J_i (\lambda_{ij} \rho) \sin(j\theta)$$

$$\phi = \phi_0 + \sum \sum a_{ij} J_i (\lambda_{ij} \rho) \cos(j\theta) + b_{ij} J_i (\lambda_{ij} \rho) \sin(j\theta)$$



- A separate set of codes written with Mathematica and Fortran
 - Could be a verification opportunity on which order is accurate enough
 - This investigation of the appropriate converging order may take long
- Two track development approaches with GTROTA
 - Accuracy version (with higher order poloidal asymmetries)
 - Plasma control version (for near-real-time calculations of all species)
 - This will have to be collaboration with PCS developers

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Ongoing Researches

- Publication plans
- Researches under investigation/collaboration

Please note that any research topics discussed in this talk can be considered as collaboration opportunities for others.

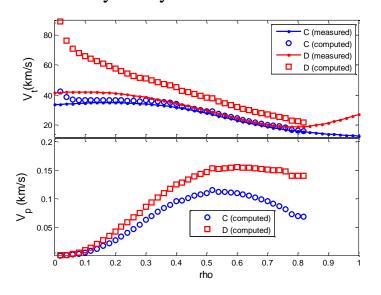


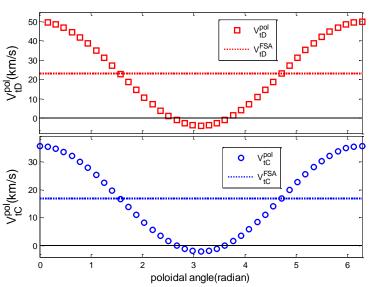


Publication Plans

- Theory verification with DIII-D deuterium velocity measurements in L-mode (2013-2015)
 - First test opportunity of the theory against deuterium measurements
 - Deuterium measurement became available from Dr. Brian Grierson
 - Below: DIII-D 145180 (1220ms)
 - Interesting findings on poloidal variation of toroidal velocities
 - Similar experimental findings with ECEI rotation images analyzed by G.S. Yoon at POSTECH







Toroidal velocities (D and C)

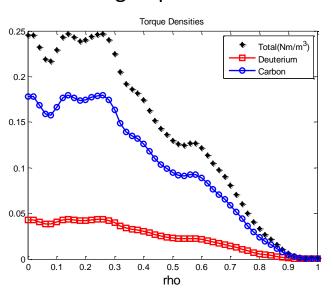
Toroidal velocity asymmetries (D and C) at rho=0.8

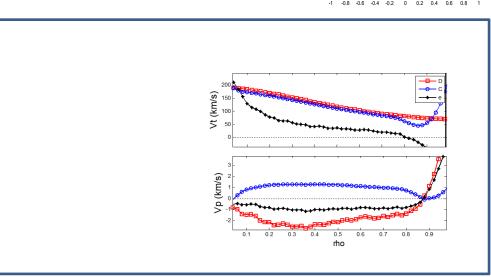


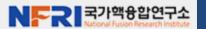


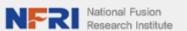
Publication Plans

- EAST and KSTAR H- and L-mode shot analyses (2015-2017)
 - Goal was to analyze more L-mode shots and upgrade GTROTA capability to calculate rotations with RF heatings
 - EAST shots with LHCD and ICRF shots
 - KSTAR L-mode shots
 - Analyses finalized
 - Need to extend GTROTA features to add HFS-LFS plotting option;
 - Planning to publish with new features available in GTROTA







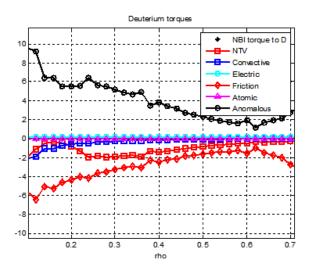


Publication Plans

- Theoretical investigation of Gyroviscous cancellation validity in Tokama plasmas (2014-2015)
 - The well-known gyroviscous cancellation in sheared slab geometry [Plasma Physics 4, 1766 (1992)] has been investigated
 - using a systematic perturbative method
 - based on the Mikhailovskii-Tsypin's closure relation
 - on the large gyrofrequency ordering for flowing plasmas
 - $O(\delta^1)$ 1st order surviving terms (in colors):

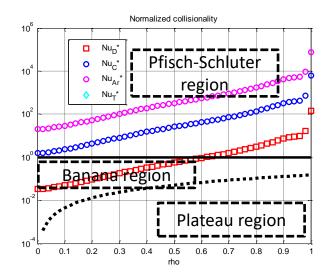
$$\partial_{t} \overrightarrow{V}_{0\parallel} + \overrightarrow{V}_{d(1)} \cdot \nabla \overrightarrow{V}_{(0)\parallel} + V_{(0)} \cdot \nabla \overrightarrow{V}_{(1)\parallel} + \frac{1}{m_{a} n_{0}} \nabla_{\parallel} p_{0} = -\Omega_{a} \left(\hat{b} \times \overrightarrow{V}_{1} \right)_{\parallel} + \frac{q_{a}}{m_{a}} \overrightarrow{E}_{0\parallel} + \frac{1}{m_{a} n_{0}} \overrightarrow{C}_{0\parallel}^{10}$$

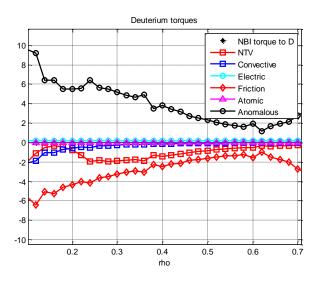
- ➤ These are the surviving terms at one order lower than investigated by Chang and Callen [Plasma Physics 4, 1766 (1992)]
 - Submitted for publication but withdrawn for numerical verifications
 - Presented at conferences





- Collisionality effects on rotation and transport
 - Motivated by too many assumptions applied in plasma researches
 - ERT retains all terms in the MBE (both toroidal and poloidal coordinates)
 - This research is to holistically understand collisionality effects
 - within a discharge: collisionality regimes change and differ for different species
 - among different discharges: Different discharges have different regime distributions
 - Q: Can I investigate collisionality effects on various shots and find any consistency on rotation/transport?
 - Will require analyses of many discharges to answer this question

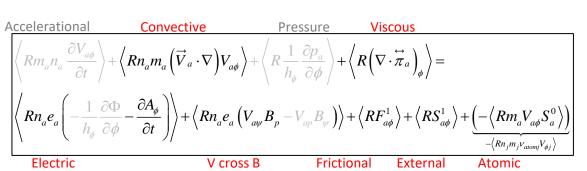


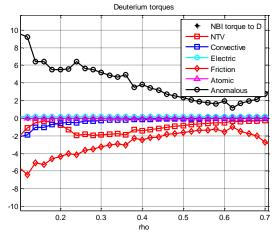






- Collisionality effects on rotation and transport
 - However, I can provide my rotation/transport calculations to others so that they can identify right assumptions based on the collisionality and apply appropriate theories
 - Calculations in toroidal angular torques are available in GTROTA





Plan to do the same code upgrade for poloidal torque balance in the future

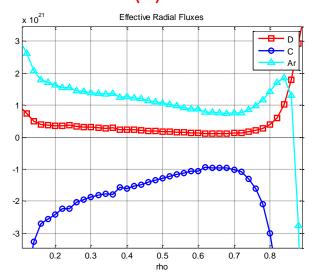
$$\left\langle rn_{j}m_{j}\left(\overrightarrow{V}_{j}\bullet\nabla\right)\overrightarrow{V}_{j\theta}\right\rangle + \left\langle \frac{r}{h_{\theta}}\frac{\partial p_{j}}{\partial\theta}\right\rangle + \left\langle r\left(\nabla\bullet\overrightarrow{\pi}_{j}\right)_{\theta}\right\rangle = \left\langle rF_{\theta j}\right\rangle + \left\langle rn_{j}e_{j}E_{\theta}\right\rangle - \left\langle rn_{j}e_{j}V_{rj}B_{\phi}\right\rangle + \left\langle rS_{\theta j}^{1}\right\rangle - \left\langle rm_{j}V_{\theta j}S_{j}^{0}\right\rangle$$



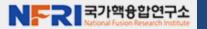


- A coupled study of ion species rotation and transport
 - Motivation is to compare radial transport (Vr and flux) calculations from GTROTA to other calculations based on diffusivity coefficients
 - Collaboration with KAIST team on Ar transport study (2015 2017)
 - with density gradient: introduces "Diffusion coefficient (D)"
 - with convective effect: introduces "Convective coefficient (V)"

$$\frac{\partial n_a}{\partial t} + \underbrace{\nabla \cdot \left(n_a \overrightarrow{V}_a \right)}_{\mathbf{D}fn(n_a) + \mathbf{V}fn(\overrightarrow{V}_a)} = S_a^0$$

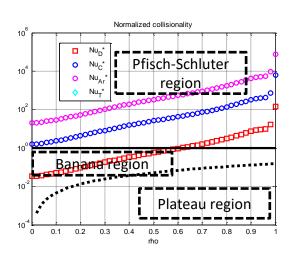


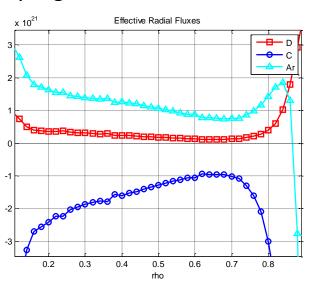
- KSTAR experiments with Ar injection
- Analyses delayed due to the temperature measurement accuracy
- A dedicated conference slides in Appendix B



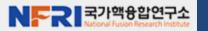


- A coupled study of ion species rotation and transport
 - Motivation: to understand heavy ion (W, Ar, or Ne) transport
 - Findings
 - Toroidal torque balance different for different ion species
 - Radial fluxes of different ion species can be opposite
 - Probably due to different collisionality regime distributions



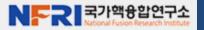


- This research effort is in preliminary stage and need some guidance
 - A dedicated talk slides in Appendix B



CONCLUSIONS

- Theoretical improvement of ERT for the edge rotation study will continue
- Computational code development will continue
- Publications to follow to report the progress and findings to the plasma physics community
- Any questions or collaboration discussions can be emailed to my permanent email below
 - yuri157@gmail.com
- Thank you for your attention!
- •Questions and Answers



Thank you for your attention!

my permanent contact: yuri157@gmail.com





APPENDIX A: GENERALIZED VISCOUS EFFECTS FOR NON-AXISYMMETRIC TOKAMAK PLASMAS

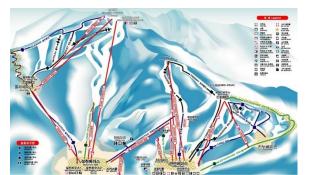
Cheonho Bae

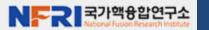
cbae@nfri.re.kr

National Fusion Research Institute, Daejeon, South Korea

January 2017
KSTAR conference, Muju resort, South Korea







Contents

- 1. NEOCLASSICAL VISCOUS TORQUES
- 2. WHERE AM I TODAY?
- 3. WHERE AM I GOING NEXT?
- 4. WHAT CAN THIS WORK DO FOR US?
- 5. QUESTIONS/DISCUSSIONS





NEOCLASSICAL VISCOUS TORQUES

- Viscous forces in tokamak coordinates
 - Toroidal and poloidal momentum balance equations

$$\begin{split} & n_{j}m_{j}\left(\overrightarrow{V}_{j}\bullet\nabla\right)\overrightarrow{V}_{j\phi} + \left(\nabla\bullet\overrightarrow{\pi}_{j}\right)_{\phi} = n_{j}e_{j}\mathbf{E}_{\phi}^{A} + n_{j}e_{j}V_{rj}B_{\theta} + F_{\phi j} + M_{\phi j} - n_{j}m_{j}V_{atom j}V_{\phi j} \\ & n_{j}m_{j}\left(\overrightarrow{V}_{j}\bullet\nabla\right)\overrightarrow{V}_{j\theta} + \frac{1}{h_{\theta}}\frac{\partial p_{j}}{\partial\theta} + \left(\nabla\bullet\overrightarrow{\pi}_{j}\right)_{\theta} = F_{\theta j} + n_{j}e_{j}E_{\theta} - n_{j}e_{j}V_{rj}B_{\phi} + S_{\theta j}^{1} - m_{j}V_{\theta j}S_{j}^{0} \end{split}$$

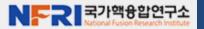
Viscosity evolution equation

$$-\Omega_{a}\left(\overrightarrow{\pi}_{a}\times\widehat{b}-\widehat{b}\times\overrightarrow{\pi}_{a}\right) = \underbrace{2p_{a}\overrightarrow{\nabla V_{a}} + \frac{4}{5}\overrightarrow{\nabla q_{a}}}_{\textit{Mikhailovski-Tsypin}} + \underbrace{\partial_{t}\overrightarrow{\pi}_{a} + \left(\overrightarrow{V}_{a}\cdot\nabla\overrightarrow{\pi}_{a} + \nabla\cdot\overrightarrow{V}_{a}\overrightarrow{\pi}_{a}\right) + 2\overrightarrow{\pi}_{a}\cdot\nabla\overrightarrow{V}_{a} + \nabla\cdot\overrightarrow{\sigma}_{a} - \overrightarrow{C}_{a}}_{\textit{Mikhailovski-Tsypin}}$$

Braginskii's viscosity representation

$$\overrightarrow{\pi}_{\alpha\beta} = \underbrace{\left(-\eta_{0}W_{\alpha\beta}^{0}\right)}_{\pi_{\alpha\beta}^{0}} + \underbrace{\left(\eta_{3}W_{\alpha\beta}^{3} + \eta_{4}W_{\alpha\beta}^{4}\right)}_{\pi_{\alpha\beta}^{34}} - \underbrace{\left(\eta_{1}W_{\alpha\beta}^{1} + \eta_{2}W_{\alpha\beta}^{2}\right)}_{\pi_{\alpha\beta}^{12}} = \underbrace{\pi_{\alpha\beta}^{0}}_{\text{wiscosity}} + \underbrace{\pi_{\alpha\beta}^{34}}_{\text{gyroviscosity}} + \underbrace{\pi_{\alpha\beta}^{12}}_{\text{viscosity}}$$

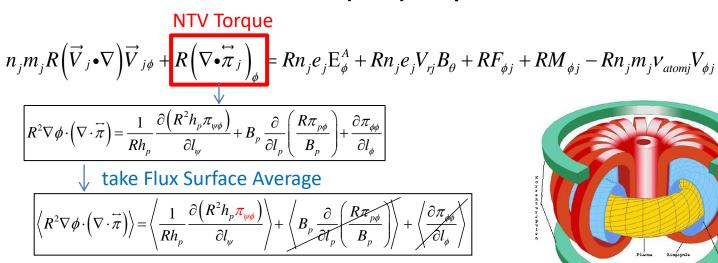
$$\text{where} \qquad \underbrace{\eta_{0} = 0.96nT\tau}_{\text{gy}} >> \underbrace{\eta_{3} = \frac{1}{2}\frac{nT}{\Omega}}_{\text{gy}}, \quad \eta_{4} = 2\eta_{3} >> \underbrace{\eta_{1} = \frac{3}{10}\frac{nT}{\Omega^{2}\tau}}_{\text{gy}}, \quad \eta_{2} = 4\eta_{1} \qquad \Rightarrow \underbrace{\left(\eta_{0} >> \eta_{3,4} >> \eta_{1,2}\right)}_{\text{gy}} + \underbrace{\left(\eta_{0} >> \eta_{1,2}\right)}_{\text{gy}} + \underbrace{\left(\eta$$





What do I mean by "Generalized Viscous Effects"?

Neoclassical Toroidal Viscous (NTV) torque



Neoclassical Poloidal Viscous (NPV) torque

$$rn_{j}m_{j}\left(\overrightarrow{V}_{j}\bullet\nabla\right)\overrightarrow{V}_{j\theta} + \frac{r}{h_{\theta}}\frac{\partial p_{j}}{\partial\theta} + r\left(\nabla\bullet\overrightarrow{\pi}_{j}\right)_{\theta} = rF_{\theta j} + rn_{j}e_{j}E_{\theta} - rn_{j}e_{j}V_{rj}B_{\phi} + rS_{\theta j}^{1} - rm_{j}V_{\theta j}S_{j}^{0}$$

$$\boxed{ \left(\nabla\cdot\overrightarrow{\pi}\right)_{p} = \sum_{j=\psi,p,\phi} \left[\frac{1}{h_{\psi}h_{p}h_{\phi}}\frac{\partial}{\partial j}\left(\frac{\left(h_{\psi}h_{p}h_{\phi}\right)\pi_{jp}}{h_{j}}\right) + \sum_{k=\psi,p,\phi}\Gamma_{pk}^{j}\pi_{jk}\right] }$$

Work out NTV and NPV torques in non-axisymmetric plasmas





Flux Surface Averaged NTV torque in non-axisymmetric plasmas

$$\begin{split} & N \mathsf{TV} \, \mathsf{Torque} \\ & n_j m_j \left\langle \left[R \Big(\overrightarrow{V}_j \bullet \nabla \Big) \overrightarrow{V}_j \right]_{\phi} \right\rangle + \left\langle \left[R \Big(\nabla \bullet \overrightarrow{\pi}_j \Big) \right]_{\phi} \right\rangle = \left\langle R n_j e_j \mathbf{E}_{\phi}^A \right\rangle + \left\langle R n_j e_j V_{rj} B_{\theta} \right\rangle + \left\langle R F_{\phi j} \right\rangle + \left\langle R M_{\phi j} \right\rangle - \left\langle R n_j m_j V_{atomj} V_{\phi j} \right\rangle \\ & \left\langle R^2 \nabla \phi \cdot \left(\nabla \cdot \overrightarrow{\pi} \right) \right\rangle = \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi} \right)}{\partial l_{\psi}} \right\rangle \\ & \left\langle R^2 \nabla \phi \cdot \left(\nabla \cdot \overrightarrow{\pi} \right) \right\rangle = \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^0 \right)}{\partial l_{\psi}} \right\rangle + \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle + \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle + \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{1}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R^2 h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{\partial \left(R h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} \right\rangle \\ & = \frac{2}{R h_p} \frac{\partial \left(R h_p \pi_{\psi \phi}^{34} \right)}{\partial l_{\psi}} + \left\langle \frac{R h_p}{R h_p} \frac{$$

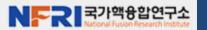
$$\pi_{\psi\phi} \approx \pi_{\psi\phi}^{0} + \pi_{\psi\phi}^{34} = -\eta_{0}W_{\psi\phi}^{0} + \eta_{4}\left(\frac{1}{2}W_{\psi\phi}^{3} + W_{\psi\phi}^{4}\right) - \eta_{2}\left(\frac{1}{4}W_{\psi\phi}^{1} + W_{\psi\phi}^{2}\right)$$

$$W_{\psi\phi}^{0} = \frac{3}{2}\int_{\psi}^{\infty} f_{p}H^{0}$$

$$f_{\psi} \equiv \frac{B_{\psi}}{B}, \quad f_{p} \equiv \frac{B_{p}}{B}, \quad f_{\phi} \equiv \frac{B_{\phi}}{B}$$

$$H^{0} = \sum_{\mu,\nu} \left(f_{\mu} f_{\nu} - \frac{1}{3} \delta_{\mu\nu} \right) W_{\mu\nu}$$

$$W_{\mu\nu} = \underbrace{\left(\overrightarrow{\nabla V} \right)_{\mu\nu}}_{\mu\nu} + \underbrace{\left(\frac{2}{5p} \overrightarrow{\nabla q} \right)_{\mu\nu}}_{Mikhailovskiii-Tsypin}$$





WHERE AM I TODAY?

- Axisymmetric NTV and NPV torques calculated in GTROTA
 - NTV represented with gyroviscosity / Braginskii's closure

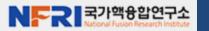
$$\left\langle R^{2}\nabla\phi\cdot\left(\nabla\cdot\overrightarrow{\pi}\right)\right\rangle = \underbrace{\left\langle\frac{1}{Rh_{p}}\frac{\partial\left(R^{2}h_{p}\pi_{\psi\phi}^{0}\right)}{\partial l_{\psi}}\right\rangle + \underbrace{\left\langle\frac{1}{Rh_{p}}\frac{\partial\left(R^{2}h_{p}\pi_{\psi\phi}^{34}\right)}{\partial l_{\psi}}\right\rangle + \underbrace{\left\langle\frac{1}{Rh_{$$

$$W_{\mu\nu} = \underbrace{\left(\overrightarrow{\nabla V}\right)_{\mu\nu}}_{\text{valid for high collisionality}} + \left(\frac{2}{5p}\overrightarrow{\nabla q}\right)_{\mu\nu}$$

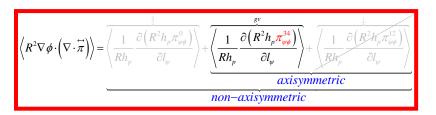
- * Two neoclassical codes that handle gyroviscosity: GTROTA / NEO
- GTROTA based on collisionality-extended Braginskii's viscosity
 - Parallel viscosity coefficient extended to low collisionality (trapped particle effect) by Shaing

$$\begin{array}{c} \text{collisionality dependent} \\ \hline \eta_0 = 0.96nT\tau \\ \hline \end{array}, \quad \begin{array}{c} \frac{1}{\eta_0} = \frac{1}{2}\frac{nT}{\Omega}, \quad \eta_4 = 2\eta_3 \\ \hline \end{array}, \quad \begin{array}{c} \eta_1 = \frac{3}{10}\frac{nT}{\Omega^2\tau}, \quad \eta_2 = 4\eta_1, \\ \hline \end{array} \\ \hline \end{array} \\ \begin{array}{c} \eta_0 \gg \eta_{3,4} \gg \eta_{1,2} \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} \\ \begin{array}{c} \bullet \\ \text{Shaing-banana plateau-PS:} \\ \hline \\ \eta_{0j} = \frac{n_j m_j V_{thj} q R_0 \varepsilon^{-\frac{3}{2}2} v_{ij}^*}{\left(1 + \varepsilon^{-3/2} v_{ij}^*\right) \left(1 + v_{ij}^*\right)} \equiv n_j m_j V_{thj} q R f_j \\ \hline \end{array}$$

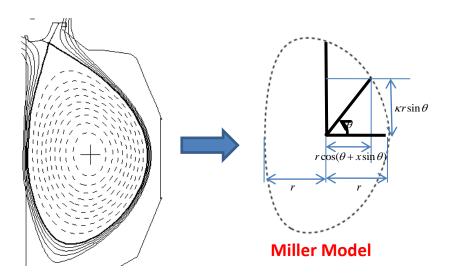
-> valid for arbitrary collisionality

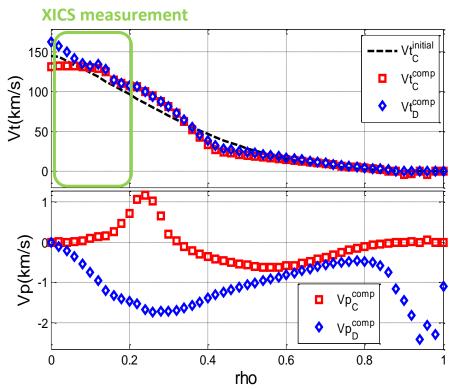


- GTROTA considers first-order poloidal asymmetries in density, velocity, and electrostatic potential
 - Rotation calculations with KSTAR #5505-2500ms (H-mode with NBI) [Bae et al., PoP 21, 012504(2014)]
 - No non-axisymmetric magnetic perturbation in this shot



$$\pi_{\psi\phi} \approx \pi_{\psi\phi}^{0} + \pi_{\psi\phi}^{34} = -\eta_{0} W_{\psi\phi}^{0} + \eta_{4} \left(\frac{1}{2} W_{\psi\phi}^{3} + W_{\psi\phi}^{4} \right) - \eta_{2} \left(\frac{1}{4} W_{\psi\phi}^{1} + W_{\psi\phi}^{2} \right)$$









WHERE AM I GOING NEXT?

- Generalize to non-axisymmetric NTV and NPV torques
 - Theoretical study published [Stacey and Bae, PoP 2015]
 - Theoretical model development for GTROTA in progress
 - Extend to Mikhailovskii-Tsypin's closure
 - need Heat Flux Density Evolution Equation

$$\left\langle R^{2} \nabla \phi \cdot \left(\nabla \cdot \overrightarrow{\pi} \right) \right\rangle = \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{0} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1}{Rh_{p}} \frac{\partial \left(R^{2}h_{p}\pi_{\psi\phi}^{34} \right)}{\partial l_{\psi}} \right\rangle}_{} + \underbrace{\left\langle \frac{1$$

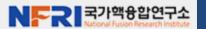
$$\boxed{\pi_{\psi\phi} \approx \pi_{\psi\phi}^0 + \pi_{\psi\phi}^{34} = -\eta_0 W_{\psi\phi}^0 + \eta_4 \left(\frac{1}{2} W_{\psi\phi}^3 + W_{\psi\phi}^4\right) - \eta_2 \left(\frac{1}{4} W_{\psi\phi}^1 + W_{\psi\phi}^2\right)}$$

$$W_{\psi\phi}^{0} = \frac{3}{2} f_{\psi} f_{p} H^{0}$$

$$W_{\mu\nu} = \left(\overline{\nabla V}\right)_{\mu\nu} + \left(\frac{2}{5p}\overline{\nabla q}\right)_{\mu\nu}$$

$$-\Omega_a(\hat{b}\times\vec{q}_a) = \frac{5}{2m_a}p_a\nabla T_a + \frac{1}{2}\overrightarrow{C}_a^{11} + O(higher)$$

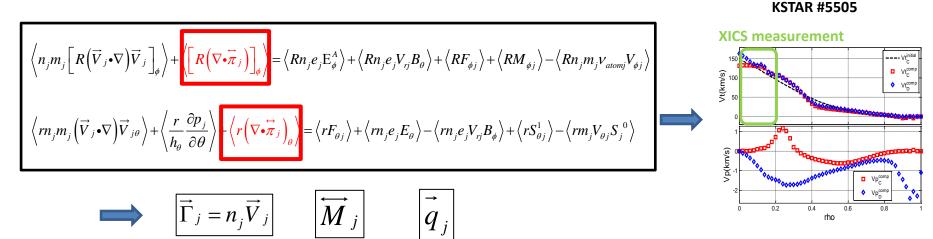
$$\begin{bmatrix} \left(f_{\psi} f_{\psi} - \frac{1}{3} \right) \left\{ \frac{4}{3} \frac{\partial V_{\psi}}{\partial l_{\psi}} - \frac{2}{3} \left(\frac{\partial V_{p}}{\partial l_{p}} + \frac{\partial V_{\phi}}{\partial l_{\phi}} \right) + 2 \left(\frac{1}{h_{\psi}} \frac{\partial h_{\psi}}{\partial l_{p}} V_{p} + \frac{1}{h_{\psi}} \frac{\partial h_{\psi}}{\partial l_{\phi}} V_{\phi} \right) \right\} + \\ \left(f_{p} f_{p} - \frac{1}{3} \right) \left\{ \frac{4}{3} \frac{\partial V_{p}}{\partial l_{p}} - \frac{2}{3} \left(\frac{\partial V_{\psi}}{\partial l_{\psi}} + \frac{\partial V_{\phi}}{\partial l_{\phi}} \right) + 2 \left(\frac{1}{h_{p}} \frac{\partial h_{p}}{\partial l_{\psi}} V_{\psi} + \frac{1}{h_{p}} \frac{\partial h_{p}}{\partial l_{\phi}} V_{\phi} \right) \right\} + \\ \left(f_{\phi} f_{\phi} - \frac{1}{3} \right) \left\{ \frac{4}{3} \frac{\partial V_{\phi}}{\partial l_{\phi}} - \frac{2}{3} \left(\frac{\partial V_{p}}{\partial l_{p}} + \frac{\partial V_{\psi}}{\partial l_{\psi}} \right) + 2 \left(\frac{1}{h_{\phi}} \frac{\partial h_{\phi}}{\partial l_{\psi}} V_{\psi} + \frac{1}{h_{\phi}} \frac{\partial h_{\phi}}{\partial l_{p}} V_{p} \right) \right\} + \\ 2 f_{\psi} f_{p} \left\{ \frac{\partial V_{\psi}}{\partial l_{p}} + \frac{\partial V_{p}}{\partial l_{\psi}} - \left(\frac{1}{h_{p}} \frac{\partial h_{p}}{\partial l_{\phi}} V_{p} + \frac{1}{h_{\phi}} \frac{\partial h_{\phi}}{\partial l_{p}} V_{\phi} \right) \right\} + \\ 2 f_{p} f_{\phi} \left\{ \frac{\partial V_{\phi}}{\partial l_{\psi}} + \frac{\partial V_{p}}{\partial l_{\phi}} - \left(\frac{1}{h_{p}} \frac{\partial h_{p}}{\partial l_{\phi}} V_{p} + \frac{1}{h_{\phi}} \frac{\partial h_{\phi}}{\partial l_{p}} V_{\phi} \right) \right\} + \\ 2 f_{\psi} f_{\phi} \left\{ \frac{\partial V_{\phi}}{\partial l_{\psi}} + \frac{\partial V_{\psi}}{\partial l_{\phi}} - \left(\frac{1}{h_{\psi}} \frac{\partial h_{\psi}}{\partial l_{\phi}} V_{\psi} + \frac{1}{h_{\phi}} \frac{\partial h_{\phi}}{\partial l_{\psi}} V_{\phi} \right) \right\}$$





WHAT CAN THIS WORK DO FOR US?

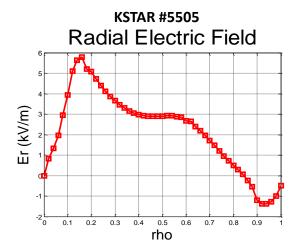
- Rotation and transport predictions of deuterium for nonaxisymmetric plasmas
 - Generalized viscous contribution to rotation and transport



Increase accuracy in Er calculation

$$\overline{E}_{r} = -V_{thj}\overline{B}_{\theta} \left[V_{\theta j} \frac{\left\langle \frac{1}{1 + \varepsilon \cos \xi} \right\rangle}{\left\langle \frac{1}{h_{r}} \right\rangle} - V_{\phi j} \left(1 + \frac{\partial R_{0}(r)}{\partial r} \right) \frac{\left\langle \frac{1}{(1 + \varepsilon \cos \xi)} \frac{1}{h_{r}} \right\rangle}{\left\langle \frac{1}{h_{r}} \right\rangle} - \frac{1}{V_{thj}} \frac{1}{\overline{n}_{j} e_{j} \overline{B}_{\theta}} \frac{\partial \overline{P}_{j}}{\partial r} \right]$$

$$\left[\overline{E}_r^{cir} = \frac{1}{\overline{n_j} e_j} \frac{\partial \overline{P}_j}{\partial r} - \left[V_{\theta j} \overline{B}_{\phi} - V_{\phi j} \overline{B}_{\theta} \right] \right]$$





Thank you for your attention! Questions/Comments







APPENDIX B: A COUPLED STUDY OF PLASMA ROTATION AND TRANSPORT

: COMPARISON OF TOROIDAL TORQUE CONTRIBUTIONS
IN AXISYMMETRIC TOKAMAK PLASMAS

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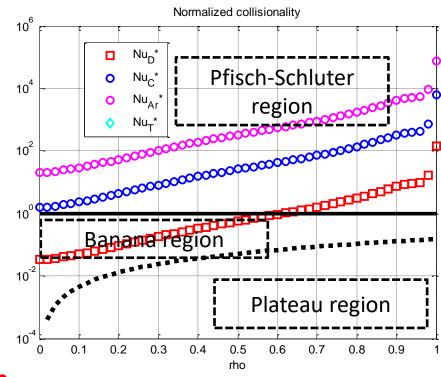
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11 July 2017 Sapporo, Japan

INTRODUCTION

Practical questions!!!

- 1. How are the rotation & transport differ for different ion species?
 - considering their collisionalities differ significantly
- 2. Can we predict deuterium rotation & transport from the impurity measurements?
 - Most transport studies done with impurities
- 3. Which physical term(s) in the angular momentum(or torque) balance equation have the largest contributions to rotation & transport?



$$\left[\left\langle Rm_{a}n_{a}\frac{\partial V_{a\phi}}{\partial t} \right\rangle + \left\langle Rn_{a}m_{a}\left(\overrightarrow{V}_{a} \bullet \nabla\right)V_{a\phi} \right\rangle + \left\langle R\frac{1}{h_{\phi}}\frac{\partial p_{a}}{\partial \phi} \right\rangle + \left\langle R\left(\nabla \bullet \overrightarrow{\pi}_{a}\right)_{\phi} \right\rangle = \left\langle Rn_{a}e_{a}\left(-\frac{1}{h_{\phi}}\frac{\partial \Phi}{\partial \phi} - \frac{\partial A_{\phi}}{\partial t}\right) \right\rangle + \left\langle Rn_{a}e_{a}\left(\overrightarrow{V}_{ar}B_{\theta} - V_{a\theta}B_{r}\right) \right\rangle + \left\langle RF_{a\phi}^{1} \right\rangle + \left\langle RS_{a\phi}^{1} \right\rangle - \left\langle Rm_{a}V_{a\phi}S_{a}^{0} \right\rangle + \left\langle RS_{a\phi}^{1} \right\rangle + \left\langle RS_{a\phi$$

A coupled study of plasma rotation & transport to answer these questions!

INTRODUCTION

- Most common "particle (or flux)" transport study of today
 - is based on the continuity equation

$$\frac{\partial n_a}{\partial t} + \underbrace{\nabla \cdot \left(n_a \overrightarrow{V}_a \right)}_{\text{D}fn(n_a) + \text{V}fn(\overrightarrow{V}_a)} = S_a^0$$

- closes this equation with approximated physical effects
 - with density gradient: introduces "Diffusion coefficient (D)"
 - with convective effect: introduces "Convective coefficient (V)"
- We use this model to study individual ion transport
 - by injecting a specific impurities (Ar, Ne, W, etc.)
 - measure fluxes to determine D, V, and effective Vr
 - require dedicated injection systems and diagnostics
 - but still study transport of only one impurity
- Q: can't we just solve the momentum balance equation(MBE) to get Vr?
 - as we do in plasma rotation study

$$n_{a}m_{a}\frac{\partial \overrightarrow{V}_{a}}{\partial t}+n_{a}m_{a}\left(\overrightarrow{V}_{a}\bullet\nabla\right)\overrightarrow{V}_{a}+\nabla p_{a}+\nabla\bullet\overrightarrow{\pi}_{a}=n_{a}e_{a}\left(\overrightarrow{E}+\overrightarrow{V}_{a}\times\overrightarrow{B}\right)+\overrightarrow{F}_{a}^{1}+\left(\overrightarrow{S}^{1}-m_{a}\overrightarrow{V}_{a}S^{0}\right)$$

INTRODUCTION

Plasma rotation study solves MBE to get velocities (mostly Vt and Vp)

$$n_{a}m_{a}\frac{\partial \overrightarrow{V}_{a}}{\partial t}+n_{a}m_{a}\left(\overrightarrow{V}_{a}\bullet\nabla\right)\overrightarrow{V}_{a}+\nabla P_{a}+\nabla\bullet\overrightarrow{\pi}_{a}=n_{a}e_{a}\left(\overrightarrow{E}+\overrightarrow{V}_{a}\times\overrightarrow{B}\right)+\overrightarrow{F}_{a}^{1}+\left(\overrightarrow{S}^{1}-m_{a}\overrightarrow{V}_{a}S^{0}\right)$$

- if Vr can also be calculated, two studies can be coupled
- Plasma rotation theories are developed by two main approaches
 - NCLASS (based on Hirshman-Sigmar approach) [Houlberg et. al., 1998]
 - most famous with two (parallel and perpendicular) MBEs to calculate neoclassical rotations of multi-ions
 - but no further development to couple it with particle/heat transport
 - GTROTA (based on Stacey-Sigmar approach) [Bae et. al., Comp. Phys. Comm. 2013]
 - introduced as "Extended Plasma Rotation Theory (EPRT)" [Bae et. el., NF 2013]
 - takes MBE in three coordinates (radial, poloidal, toroidal)
 - direct comparison with Vt and Vp measurements possible
 - Vr specifically appears in the toroidal torque balance (next slide)
- Q: can I extend EPRT/GTROTA to find Vr/radial fluxes for most generalized tokamak plasmas(both axisymmetric and non-axisymmetric)?



CURRENT STATUS OF EPRT/GTROTA

- Retains all the terms in the MBEs and in three coordinates
- Velocity calculation models for Vt and Vp
 - Toroidal direction: toroidal torque balance (Flux Surface Averaged)
 - Vr appears in this equation => Effective Vr (later slide)

$$\left| \left\langle Rm_{a}n_{a}\frac{\partial V_{a\phi}}{\partial t} \right\rangle + \left\langle Rn_{a}m_{a}\left(\overrightarrow{V}_{a}\bullet\nabla\right)V_{a\phi} \right\rangle + \left\langle R\frac{1}{h_{\phi}}\frac{\partial p_{a}}{\partial \phi} \right\rangle + \left\langle R\left(\nabla\bullet\overrightarrow{\pi}_{a}\right)_{\phi} \right\rangle = \\ \left\langle Rn_{a}e_{a}\left(-\frac{1}{h_{\phi}}\frac{\partial\Phi}{\partial\phi} - \frac{\partial A_{\phi}}{\partial t}\right) \right\rangle + \left\langle Rn_{a}e_{a}\left(V_{ar}B_{\theta} - V_{a\theta}B_{r}\right) \right\rangle + \left\langle RF_{a\phi}^{1} \right\rangle + \left\langle RS_{a\phi}^{1} \right\rangle - \left\langle Rm_{a}V_{a\phi}S_{a}^{0} \right\rangle$$

Poloidal direction: poloidal torque balance (Flux Surface Averaged)

$$\left\langle rm_{a}n_{a}\frac{\partial V_{a\theta}}{\partial t}\right\rangle + \left\langle rn_{a}m_{a}\left(\overrightarrow{V}_{a}\bullet\nabla\right)V_{a\theta}\right\rangle + \left\langle r\frac{1}{h_{\theta}}\frac{\partial p_{a}}{\partial \theta}\right\rangle + \left\langle r\left[\nabla\bullet\overset{\leftrightarrow}{\pi}_{a}\right]_{\theta}\right\rangle =$$

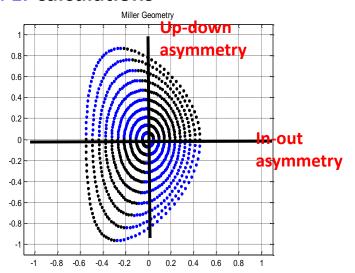
$$\left\langle r\left(-\frac{1}{h_{p}}\frac{\partial\Phi}{\partial\theta} - \frac{\partial A_{\theta}}{\partial t}\right)\right\rangle + \left\langle rn_{a}e_{a}\left(V_{a\phi}B_{r} - V_{ar}B_{\phi}\right)\right\rangle + \left\langle rF_{a\theta}^{1}\right\rangle + \left\langle r\left(S_{a\theta}^{1} - m_{a}V_{a\theta}S_{a}^{0}\right)\right\rangle$$

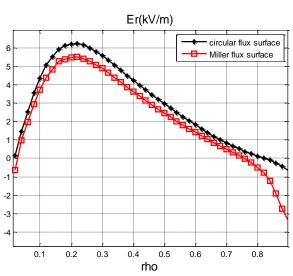
- Radial direction: radial MBE
 - Provides coupling relations with the continuity equation
 - Used to calculate 1st order poloidal variations (aka poloidal asymmetries)
 - in density, velocity, and electrostatic potential [Bae et. Al., PoP 2014]



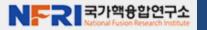
CURRENT STATUS OF EPRT/GTROTA

- Other EPRT/GTROTA features
 - Uses Miller flux surface geometry
 - for higher accuracy in velocities(Vt, Vp, Vr), momentum/particle transport, and Er calculations





- All plasma parameters (Vt, Vp, Vr, Er, etc.) self-consistently iterated
 - Maximizes the advantages of plasma fluid equations
- Calculates Er, Vt, Vp, Vr, and poloidal asymmetries of up to four ion species & electron
 - Simulation of multi-ion plasmas possible (eg., D-T plasma with Tungsten)





CURRENT STATUS OF EPRT/GTROTA

- **Current GTROTA handles axisymmetric plasmas only**
 - Non-axisymmetric theory available [Stacey and Bae, PoP 2015] but not in GTROTA
- Today, calculated Effective Vr in axisymmetric plasmas
 - defined to represent all anomalous terms (2nd order and higher / turbulence inclusive)
 - Assuming most anomalous transports are in radial direction

Convective Accelerational Pressure Viscous (NTV)

$$\left\langle Rm_{a}n_{a}\frac{\partial V_{a\phi}}{\partial t}\right\rangle + \left\langle Rn_{a}m_{a}\left(\overrightarrow{V}_{a}\bullet\nabla\right)V_{a\phi}\right\rangle + \left\langle R\frac{1}{h_{\phi}}\frac{\partial p_{a}}{\partial \phi}\right\rangle + \left\langle R\left(\nabla\bullet\overrightarrow{\pi}_{a}\right)_{\phi}\right\rangle = \\ \left\langle Rn_{a}e_{a}\left(-\frac{1}{h_{\phi}}\frac{\partial\Phi}{\partial\phi} - \frac{\partial A_{\phi}}{\partial t}\right)\right\rangle + \left\langle Rn_{a}e_{a}\left(V_{ar}B_{\theta} - V_{a\theta p}B_{r}\right)\right\rangle + \left\langle RF_{a\phi}^{1}\right\rangle + \left\langle RS_{a\phi}^{1}\right\rangle - \left\langle Rm_{a}V_{a\phi}S_{a}^{0}\right\rangle$$

Electric

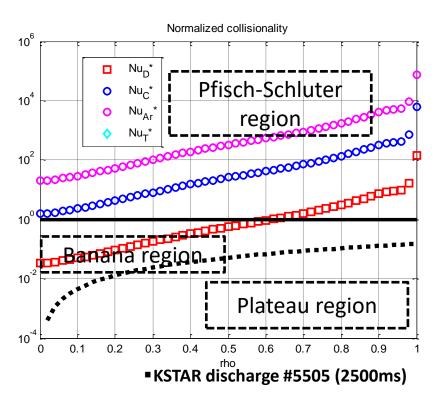
V cross B

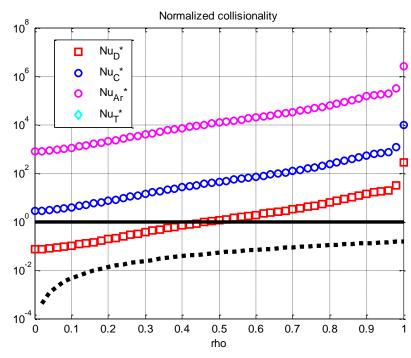
Frictional External Atomic



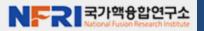
DISCHARGES ANALYZED

- Four KSTAR discharges analyzed
 - Two H-modes with Ar rotation measured (#5505-2500ms / #5953-2500ms)
 - Two L-modes with Carbon rotation measured (#13728-4500ms / 13728-4950ms)
- One simulation with Tungsten
 - Based on #5505-2500ms



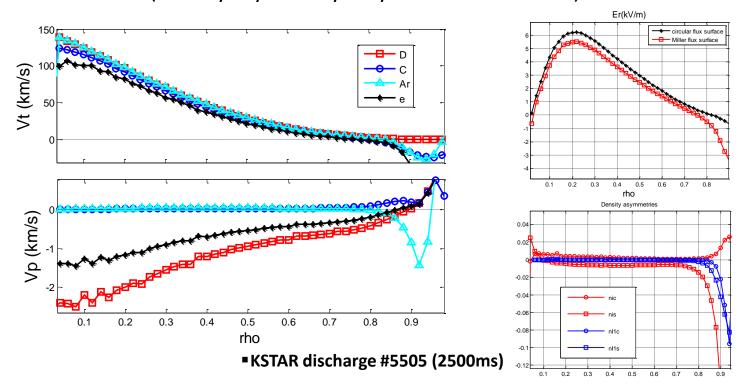


KSTAR discharge #5505 (2500ms) with Ar replace with W



ANALYSIS RESULTS

- Rotation, poloidal asymmetries, & Er (KSTAR #5505-2500ms)
 - **Vp and Vt of all ions/electrons calculated**
 - Vt very close to each other and stays within 10% of the measurement [Bae et. et., NF 2013 / Bae et. al., PoP 2014]
 - Er self-consistently(iteratively) calculated
 - Poloidal asymmetries (of density, velocity, and electrostatic potential) calculated (density asymmetry only shown in this slide)

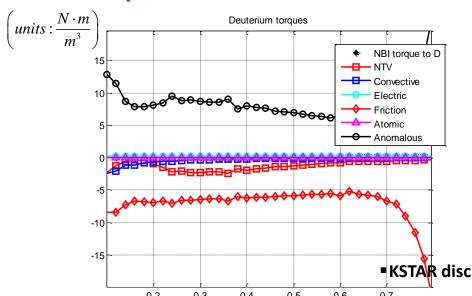


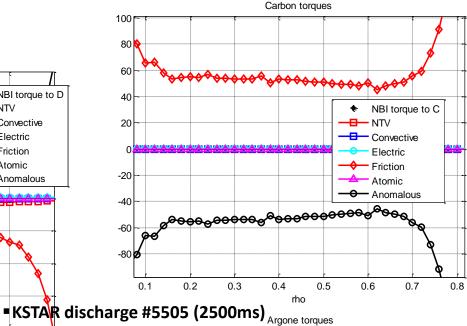




TOROIDAL TORQUE COMPARISONS

Toroidal torque densities



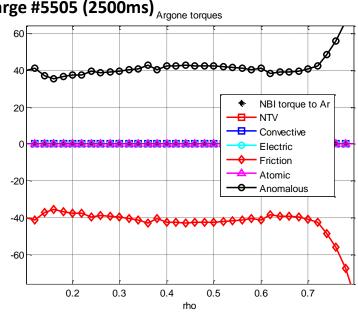


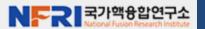
Findings

 torque balance mostly maintained by friction and anomalous ("Effective Vr cross B")

rho

- especially for the impurities
- Different balancing relations for different ion species
 - Q: Is "collisionality" most dominant effect in the rotation and transport of different ion species?

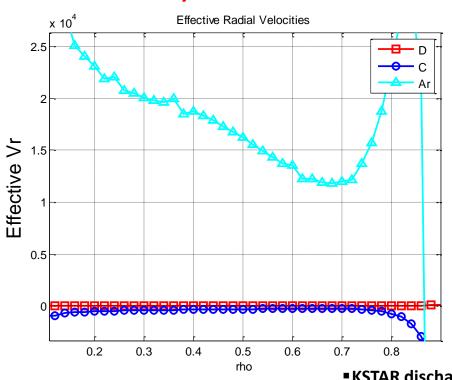


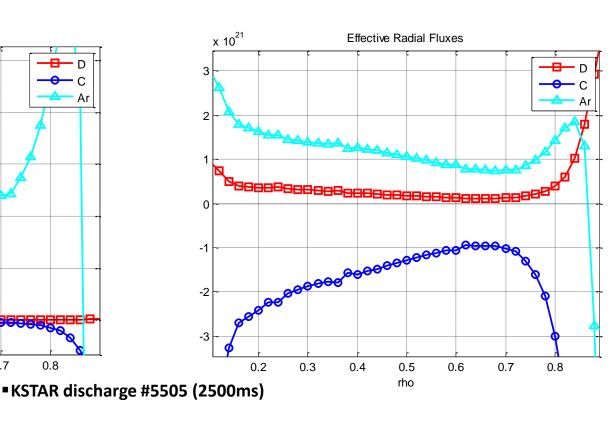




TOROIDAL TORQUE COMPARISONS

Effective Vr / radial fluxes





Findings

- Effective Vr & radial fluxes larger for impurities
- Impurity fluxes in opposite directions
 - Q: Do all the impurities always accumulate in the core?
 - Q: What physical mechanism determines these directions? Collisionality?



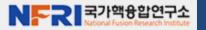
FUTURE OPPORTUNITIES

- Theoretical model development (in progress)
 - develop a separate Vr subsystem
 - develop numerical models for non-axisymmetric plasmas [Stacey and Bae, PoP 2015]
 - include 1st order toroidal variations (in Bt and others)
- Numerical code development
 - Code in non-axisymmetric theories
 - compare all the poloidal torque density terms

$$\left\langle rm_{a}n_{a}\frac{\partial V_{ap}}{\partial t}\right\rangle + \left\langle rn_{a}m_{a}\left(\overrightarrow{V}_{a}\bullet\nabla\right)V_{ap}\right\rangle + \left\langle r\frac{1}{h_{p}}\frac{\partial p_{a}}{\partial p}\right\rangle + \left\langle r\left[\nabla\bullet\overset{\leftrightarrow}{\pi}_{a}\right]_{p}\right\rangle =$$

$$\left\langle r\left(-\frac{1}{h_{p}}\frac{\partial\Phi}{\partial p} - \frac{\partial A_{p}}{\partial t}\right)\right\rangle + \left\langle rn_{a}e_{a}\left(V_{a\phi}B_{\psi} - V_{a\psi}B_{\phi}\right)\right\rangle + \left\langle rF_{ap}^{1}\right\rangle + \left\langle r\left(S_{ap}^{1} - m_{a}V_{ap}S_{a}^{0}\right)\right\rangle$$

- Simulations/analysis of modern tokamaks
 - Analyze KSTAR discharges with ITBs
 - Various modes/devices: DIII-D, KSTAR, EAST, etc.
 - W transport simulations
 - ITER-relevant simulations (ITER shapes / D-T fusion / etc.)
 - Collisionality effect simulations
- open to comments and ideas!



Thank you for your attention!

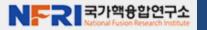
Additional slides with more details...

- 1. Simulation with Tungsten
- 2. L-mode discharge analysis results
- 3. Extended Plasma Rotation theory

my permanent contact: yuri157@gmail.com



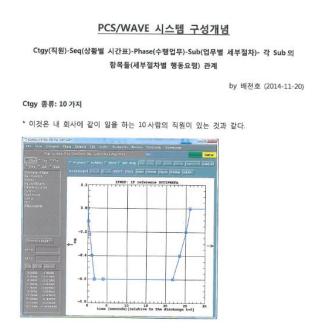
APPENDIX C: EXPERIMENTAL EXPERIENCES WITH KSTAR

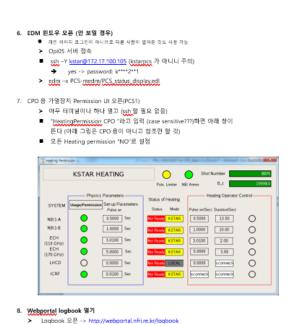




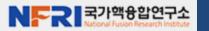
KSTAR CAMPAIGN EXPERIENCES

- CPO/APO experiences [2013-1017]
 - Served as CPO/APO for five consecutive KSTAR annual campaigns
 - Develop CPO/APO training manual and checklist



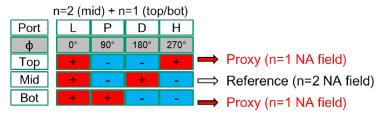


- KSTAR campaign & PAC coordinator [2013-2014]
- Rotation/Transport study shots
 - KSTAR shots with CES/XICS/ECEI diagnostics collected



PUBLICATIONS ON EXPERIMENTS

- H.H. Lee et al., *Tearing modes induced by perpendicular electron cyclotron resonance heating in the KSTAR tokamak*, Nucl. Fusion 54(2014), 103008
- Jayhyun Kim et al., *Suppression of edge localized mode crashes by multi-spectral non-axisymmetric fields in the KSTAR*, Nucl. Fusion 57, 022001 (2017)
 - Type-I ELM crash suppression reproduced both consistent and inconsistent suppression performances when compared to the DIII-D results
 - indicates a dependency of ELM suppression on the heating level and the associated kink-like plasma responses
 - dominant and malign kink-like plasma responses over the benign gap filling effects



$$\vec{B}_{\text{total}} = \vec{B}_{\text{eq}} + \vec{B}_{\text{int}} + \vec{B}_{\text{RMP}} + \vec{B}_{\text{pr}}$$

$$\overrightarrow{B}_{\mathrm{pr}} = \overrightarrow{B}_{\mathrm{pr}}^{\mathrm{lin}} + \overrightarrow{B}_{\mathrm{pr}}^{\mathrm{nl}}$$

